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COM205T Discrete Structures for Computing
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Problem Session - Logic

1. Translate each of the following sentences into FOL.

   (a) `Not all cars have carburetors`
   $\neg\forall x\ [car(x) \rightarrow carburetors(x)]$
   $\exists x\ [car(x) \wedge \neg carburetors(x)]$

   (b) `Some people are either religious or pious`
   $\exists x\ (R(x) \oplus P(x)) \equiv \exists x\ \neg(R(x) \leftrightarrow P(x))$
   $\neg\forall x\ [R(x) \leftrightarrow P(x)]$
   $\exists x((R(x) \wedge \neg P(x)) \vee (\neg R(x) \wedge P(x))$

   (c) `No dogs are intelligent`
   $\forall x\ (dog(x) \rightarrow \neg Intelligent(x))$
   $\neg\exists x\ (dog(x) \wedge Intelligent(x))$

   (d) `All babies are illogical`
   $\forall x\ (baby(x) \rightarrow illogical(x))$
   $\neg\exists x\ (baby(x) \wedge \neg illogical(x))$

   (e) `Every number is either negative or has a square root`
   $\forall x\ \neg(number(x) \leftrightarrow sqroot(x))$
   $\neg\exists x\ (number(x) \leftrightarrow sqroot(x))$
   $\forall x((number(x) \wedge \neg sqroot(x)) \vee (\neg number(x) \wedge sqroot(x))$

   (f) `Some numbers are not real`
   $\exists x \neg Real(x)$ or $\neg\forall x\ Real(x)$

   (g) `Every connected and circuit-free graph is a tree`
   $\forall x\ [(conn(x) \wedge \neg cir(x)) \rightarrow tree(x)]$
   $\neg\exists x\ [(conn(x) \wedge \neg cir(x)) \wedge \neg tree(x)]$

   (h) `Not every graph is connected`
   $\neg\forall x\ connected(x)$ or $\exists x\ \neg connected(x)$

   (i) `All that glitters is not gold`
   $\forall x\ [glitter(x) \rightarrow \neg gold(x)]$
   $\neg\exists x\ [glitter(x) \wedge gold(x)]$

   (j) `There is a barber who shaves all men in the town who do not shave themselves`
   $\exists x\ [Barber(x) \wedge \forall y\ [man(y) \wedge \neg shaves(y,y)] \rightarrow shaves(x,y)]$

(k) `There is no business like show business`
$\forall x\ [(business(x) \wedge (x \neq show\ business)) \rightarrow \neg like(x, show\ business)]$

2. Rewrite each proposition symbolically, when universe of discourse is a set of real numbers

   (a) `For each integer x, there exist an integer y such that` $x + y = 0$
   $\forall x\ [int(x) \rightarrow \exists y\ (int(y) \wedge (x + y = 0))]$

   (b) `There exist an integer x such that` $x + y = y$ `for every integer y`
   $\exists x\ [int(x) \wedge \forall y\ (int(y) \rightarrow (x + y = y))]$

   (c) `For all integers x and y,` $x.y = y.x$
   $\forall x\ \forall y\ [[int(x) \wedge int(y)] \rightarrow x.y = y.x]$

   (d) `There are integers x and y such that x+y=5`
   $\exists x\ \exists y\ [(int(x) \wedge int(y)) \wedge (x + y = 5)]$

3. Using FOL, express the following

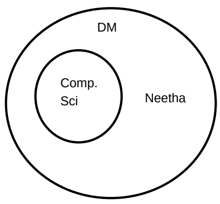
   (a) `Every student in this class has taken exactly two mathematics course at this school`
   $\forall x\ \mathbf{[}stud(x) \rightarrow \exists y_1\ \exists y_2\ \mathbf{(}y_1 \neq y_2 \wedge math(y_1) \wedge math(y_2) \wedge taken(x, y_1) \wedge taken(x, y_2) \wedge$
   $\forall y_3\ \mathbf{(}y_3 \neq y_1 \wedge y_3 \neq y_2 \wedge math(y_3) \rightarrow \neg taken(x, y_3)\mathbf{)}\ \mathbf{)]}$

   $\forall x\ (stud(x) \rightarrow mathcount(x) = 2)$

   (b) `Someone has visited every country in the world except Libya`
   $\exists x\ \forall y\ [notvisited(x, y) \leftrightarrow (y = Libya)]$
   $\exists x\ \exists! y\ [notvisited(x, y) \leftrightarrow (y = Libya)]$
   $\exists x\ [person(x) \wedge \forall y\ (country(y) \rightarrow (visited(x, y) \leftrightarrow (y \neq Libya)))]$

   (c) `No one has climbed every mountain in the Himalayas`
   $\forall x\ (person(x) \rightarrow \neg\ \forall\ y(Mountain(y) \rightarrow Climb(x, y))$
   $\neg\ \exists x\ (person(x) \wedge\ \forall\ y(Mountain(y) \rightarrow Climb(x, y))$
   $\forall x \exists y \neg \text{climb}(x, y)$, where climb$(x, y)$ is "x climbs y, "the domain x consists of all human beings and the domain y consists of all mountains in the world.

4. Check the validity: Every computer science student takes discrete mathematics. Neetha is taking discrete mathematics. Therefore, Neetha is a computer science student.
   The given conclusion is false. The following Venn diagram is a counter example for the given conclusion.



5. Check the validity: If it does not rain or it is not foggy then the sailing race will be held and life saving demonstrations will go on. If the sailing race is held, then the trophy will be

awarded. The trophy was not awarded. Therefore, it rained.

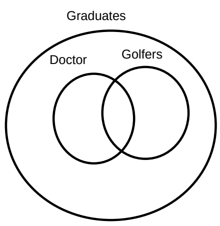
The conclusion is TRUE by the following argument.

premise	$\neg R \lor \neg F \to S \land D$	$\dots (1)$
premise	$S \to T$	$\dots (2)$
premise	$\neg T$	$\dots (3)$
1	$\neg R \lor \neg F \to S$	$\dots (4)$
$4, 2$	$\neg R \lor \neg F \to T$	$\dots (5)$
5	$\neg T \to \neg(\neg R \lor \neg F)$	$\dots (6)$
$6, 3$	$\neg(\neg R \land \neg F)$	$\dots (7)$
7	$R \land F$	$\dots (8)$
8	R	

6. Prove or Disprove: All doctors are college graduates. Some doctors are not golfers. Hence, some golfers are not college graduates.

$$\begin{array}{ll} premise: & \forall x(Doctor(x) \to Grad(x)) \\ premise: & \exists x(Doctor(x) \land \neg Golf(x)) \\ conclusion: & \exists x(Golf(x) \land \neg Grad(x)) \end{array}$$

The given conclusion is false. The following Venn diagram is a counter example for the given conclusion.



7. Using FOL: express the following

   (a) All boys in the class are at least as tall as Mr.Sharma whereas Mr.Sharma is taller than some girls in the class.

   $\forall x$ (Boys(x) $\to$ Atleast-tall(x, Sharma)) $\land \exists y$ (Girls(y)$\land$ taller(Sharma, y))

   (b) In the array A with 100 integer elements, the first fifty numbers are in increasing order and the last fifty are in decreasing order.

   $(\forall i(1 \le i \le 49)[A[i] \le A[i+1]]) \land (\forall i(51 \le i \le 99)[A[i] \ge A[i+1]])$

   (c) It is not the case that all blueline buses are bad. Some persons who drive blueline buses are not certified drivers.

   $\neg\forall x$ (Bluelinebus(x)).
   $\exists x$ (Person(x)$\land$ Drives(x, bluelinebus) $\land\neg$ Certified(x))

8. The attack will succeed only if the enemy is taken by surprise or the position is weakly defended. The enemy will not be taken by surprise unless he is overconfident. The enemy will not be overconfident if the position is weakly defended. Therefore, the attack will not succeed.

   Let, A represents attack will succeed.

E represents enemy is taken by surprise.
W represents the position is weakly defeded.
O represents he is overconfident.
The given statement can be written as follows:

$premise: \quad A \to E \vee W$
$premise: \quad \neg O \to \neg E$
$premise: \quad W \to \neg O$
$Conclusion: \neg A$

Using Truth Table, we can check whether the given conclusion follows from the premises.

						a	b	c	
A	E	W	$\neg A$	$E \vee W$	$W \wedge E$	$\neg(W \wedge E)$	$A \to (E \vee W)$	$a \wedge b$	$c \to \neg A$
T	T	T	F	T	T	F	T	F	T
T	T	F	F	T	F	T	T	T	F
T	F	T	F	T	F	T	T	T	F
T	F	F	F	F	F	T	F	F	T
F	T	T	T	T	T	F	T	F	T
F	T	F	T	T	F	T	T	T	T
F	F	T	T	T	F	T	T	T	T
F	F	F	T	F	F	T	T	T	T

The last column says that the given argument is contigency. i.e., the given argument is invalid. Thus, the conclusion does not follows from the premises.

9. Check the validity of the following implications
   (a) $P \to (Q \to R)$ equivalent to $(P \to Q) \to (P \to R)$

P	Q	R	$Q \to R$	$(P \to Q)$	$(P \to R)$	$(P \to Q) \to (P \to R)$	$P \to (Q \to R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F
T	F	T	T	F	T	T	T
T	F	F	T	F	F	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Hence, $P \to (Q \to R)$ is equivalent to $(P \to Q) \to (P \to R)$.

   (b) $[(P \to Q) \wedge (R \to S)] \to [(P \vee R) \to (Q \vee S)]$

From the table, it is clear that $[(P \to Q) \wedge (R \to S)] \to [(P \vee R) \to (Q \vee S)]$ is a contigency. Refer the table given below.

P	Q	R	S	$P \to Q$	$R \to S$	$(P \vee R)$	$(Q \vee S)$	$(P \to Q)$ $\vee$ $(R \to S)$	$(P \vee R)$ $\to$ $(Q \vee S)$	$[(P \to Q) \vee (R \to S)]$ $\to$ $[(P \vee R) \to (Q \vee S)]$
T	T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	T	T	T	T	T
T	T	F	T	T	T	T	T	T	T	T
T	F	T	T	F	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T	T	T
T	F	T	F	F	F	T	F	F	F	T
T	F	F	T	F	T	T	T	T	T	T
T	F	F	F	F	T	T	F	T	F	F
F	T	T	T	T	T	T	T	T	T	T
F	T	T	F	T	F	T	T	T	T	T
F	T	F	T	T	T	F	T	T	T	T
F	F	T	T	T	T	T	T	T	T	T
F	T	F	F	T	T	F	T	T	T	T
F	F	T	F	T	F	T	F	T	F	F
F	F	F	T	T	T	F	T	T	T	T
F	F	F	F	T	T	F	F	T	T	T

10. Show that the following propositions are valid
    (a) $[\forall x P(x) \to Q]$ equivalent to $[\exists x P(x) \to Q]$

    $\forall x P(x)$ and $\exists x P(x)$ are atomic predicates. Therefore, we can check the validity of the given proposition using truth table.
    From the last two columns we can conclude that given propositions are not equivalent.

$\forall x P(x)$	Q	$\exists x P(x)$	$\forall x P(x) \to Q$	$\exists x P(x) \to Q$
0	0	0	1	1
0	0	1	1	0
0	1	0	1	1
0	1	1	1	1
1	0	0(NA)	NA	NA
1	0	1	0	0
1	1	0(NA)	NA	NA
1	1	1	1	1

    (b) $\forall x[P \to Q(x)]$ equivalent to $[P \to \forall x Q(x)]$

    $\forall x[P \to Q(x)]$
    $\leftrightarrow \forall x[\neg P \vee Q(x)]$
    $\leftrightarrow [\neg P \vee Q(0)] \wedge [\neg P \vee Q(1)] \wedge [\neg P \vee Q(0)] \wedge \ldots$
    $\leftrightarrow [\neg P \vee (Q(0) \wedge Q(1) \wedge Q(2) \wedge \ldots)]$
    $\leftrightarrow [\neg P \vee (\forall x Q(x))]$
    Hence, the given propositions are equivalent.

11. Everyone who gets admitted into an IIT gets a job. Therefore, if there are no jobs, then nobody gets admitted into any IIT.

    Premise: $\forall x\big((\exists y(IIT(y) \wedge admit(x,y))) \to (\exists z(job(z) \wedge getjob(x,z)))\big)$
    Conclusion: $\forall z(\neg(job(z)) \to \forall x \exists y(IIT(y) \wedge \neg admit(x,y))$. (or)
    $\forall z(\neg(job(z)) \to \neg\exists x \forall y(IIT(y) \to admit(x,y))$.

12. All horses are animals. Therefore, heads (leader) of horses are heads of animals.

    Premise: $\forall x(H(x) \to A(x))$
    Conclusion: $\forall x(\exists y(H(y) \wedge Head(x,y)) \to \exists y(A(y) \wedge Head(x,y)))$.