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# Assignment 1 - Propositional Logic

Source: Discrete Mathematics and its applications, Sixth Edition - Kenneth H.Rosen
**Chapter 1, Pages 16-21**

**Question 1** *Which of these sentences are propositions? What are the truth values of those that are propositions?*

*a.* *Boston is the capital of Massachusetts.*

*b.* *Miami is the capital of Florida.*
*c.* $2 + 3 = 5$.
*d.* $5 + 7 = 10$.
*e.* $x + 2 = 11$.
*f.* *Answer this question.*

**Solution:**

a. It is a proposition and the truth value of this proposition is TRUE.
b. It is a proposition and the truth value of this proposition is FALSE
(∵ Tallahassee is the capital of Florida).

c. It is a proposition and the truth value of this proposition is TRUE.

d. It is a proposition and the truth value of this proposition is FALSE
($\because 5 + 7 = 12$).

e. It is not a proposition (∵ the given statement is neither TRUE nor FALSE unless we assign a value for the variable $x$).

f. It is not a proposition. (∵ this is an imperative sentence not a declarative sentence).

**Question 2** *Which of these are propositions? What are the truth values of those that are propositions??*

*a.* *Do not pass go.*

*b.* *What time is it?.*

*c.* *There are no black flies in Maine.*

*d.* $4 + x = 5$.

*e. The moon is made of green cheese.*

*f.* $2^n \geq 100$.

**Solution:**

a. This is not a proposition; it's a command.

b. This is not a proposition; it's a question.

c. This is a proposition that is FALSE, as anyone who has been to Maine knows.

d. This is not a proposition; its truth value depends on the value of x.

e. This is a proposition that is FALSE.

f. This is not a proposition; its truth value depends on the value of $n$.

**Question 3** *What is the negation of each of these propositions?*

*a. Today is Thursday.*

*b. There is no pollution in New Jersey.*

*c.* $2 + 1 = 3$.

*d. The summer in Maine is hot and sunny.*

**Solution:**

a. Today is not Thursday.

b. New Jersy is polluted.

c. $2+1\neq3$.

d. The summer in Maine is not hot and not sunny.

**Question 4** *Let $p$ and $q$ be the propositions*
*p: I bought a lottery ticket this week.*
*q: I won the million dollar jackpot on Friday.*
*Express each of these propositions as an English sentence.*

*a.* $\neg p$
*b.* $p \vee q$
*c.* $p \rightarrow q$

*d.* $p \wedge q$
*e.* $p \leftrightarrow q$
*f.* $\neg p \rightarrow \neg q$
*g.* $\neg p \wedge \neg q$
*h.* $\neg p \vee (p \wedge q)$

**Solution:**

a. I did not buy a lottery ticket this week.

b. Either I bought a lottery ticket this week or I won the million dollar jackpot on Friday.

c. If I bought a lottery ticket this week, then I won the million dollar jackpot on Friday.

d. I bought a lottery ticket this week and I won the million dollar jackpot on Friday.

e. I bought a lottery ticket this week if and only if I won the million dollar jackpot on Friday.

f. If I did not buy a lottery ticket this week, then I did not win the million dollar jackpot on Friday.

g. I did not buy a lottery ticket this week, and I did not win the million dollar jackpot on Friday.

h. Either I did not buy a lottery ticket this week, or else I did buy one and won the million dollar jackpot on Friday.

**Question 5** *Let $p$ and $q$ be the propositions "Swimming at the New Jersey shore is allowed"and "Sharks have been spotted near the shore,"respectively. Express each of these compound propositions as an English sentence.*

*a.* $\neg q$
*b.* $p \wedge q$
*c.* $\neg p \vee q$
*d.* $p \rightarrow \neg q$
*e.* $\neg q \rightarrow p$
*f.* $\neg p \rightarrow \neg q$
*g.* $p \leftrightarrow \neg q$
*h.* $\neg p \wedge (p \vee \neg q)$

**Solution:**

a. Sharks have not been spotted near the shore.

b. Swimming at the New Jersey shore is allowed, and sharks have been spotted near the shore.

c. Swimming at the New Jersey shore is not allowed, or sharks have been spotted near the shore.

d. If swimming at the New Jersey shore is allowed, then sharks have not been spotted near the shore.

e. If sharks have not been spotted near the shore, then swimming at the New Jersey shore is allowed.

f. If swimming at the New Jersey shore is not allowed, then sharks have not been spotted near the shore.

g. Swimming at the New Jersey shore is allowed if and only if sharks have not been spotted near the shore.

h. Swimming at the New Jersey shore is not allowed, and either swimming at the New Jersey shore is allowed or sharks have not been spotted near the shore.

**Question 6** *Let $p$ and $q$ be the propositions."The election is decided" and "The votes have been counted," respectively. Express each of these compound propositions as an English sentence.*

*a.* $\neg p$

*b.* $p \vee q$

*c.* $\neg p \wedge q$

*d.* $q \to p$

*e.* $\neg q \to \neg p$

*f.* $\neg p \to \neg q$

*g.* $p \leftrightarrow q$

*h.* $\neg q \vee (\neg p \wedge q)$

**Solution:**

a. The election is not decided.

b. The election is decided, or the votes have been counted.

c. The election is not decided, and the votes have been counted.

d. If the votes have been counted, then the election is decided.

e. If the votes have not been counted, then the election is not decided.

f. If the election is not decided, then the votes have not been counted.

g. The election is decided if and only if the votes have been counted.

h. Either the votes have not been counted, or else the election is not decided and the votes have been counted.

**Question 7** *Let $p$ and $q$ be the propositions*
$p$ *: It is below freezing.*
$q$ *: It is snowing.*
***Write these propositions using p and q and logical connectives.***

*a. It is below freezing and snowing.*

*b. It is below freezing but not snowing.*

*c. It is not below freezing and it is not snowing.*

*d. It is either snowing or below freezing (or both).*

*e. If it is below freezing, it is also snowing.*

*f. It is either below freezing or it is snowing, but it is not snowing if it is below freezing.*

*g. That it is below freezing is necessary and sufficient for it to be snowing.*

**Solution:**

a. $p \wedge q$

b. $p \wedge \neg q$

c. $\neg p \wedge \neg q$

d. $p \vee q$

e. $p \rightarrow q$

f. $(p \vee q) \wedge (p \rightarrow \neg q)$

g. $p \leftrightarrow q$

**Question 8** *Let $p$, $q$, and $r$ be the propositions*
$p$ *: You have the flu.*
$q$ *: You miss the final examination.*
$r$ *: You pass the course.*
*Express each ofthese propositions as an English sentence.*

*a.* $p \rightarrow q$

*b.* $\neg q \leftrightarrow r$

*c.* $q \rightarrow \neg r$

*d.* $p \vee q \vee r$

*e.* $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

*f.* $(p \wedge q) \vee (\neg q \wedge r)$

**Solution:**

a. If you have flu, then you miss the final examination.

b. You will pass the course if and only if don't miss the final examination.

c. If you miss the final examination, then you will not pass the course.

d. You have the flu or you miss the final examination or you pass the course.

e. If you either have the flu or miss the final examination, then you will not pass the course (or) If you have the flu, then you do not pass the course or if you miss the final examination, then you do not pass the course or both

f. Either you have the flu and miss the final examination, or you don't miss the final examination and pass the course.

**Question 9** *Let $p$ and $q$ be the propositions*
$p$ : *You drive over 65 miles per hour.*
$q$ : *You get a speeding ticket.*
*Write these propositions using $p$ and $q$ and logical connectives.*

*a. You do not drive over 65 miles per hour.*

*b. You drive over 65 miles per hour, but you do not get a speeding ticket.*

*c. You will get a speeding ticket if you drive over 65 miles per hour.*

*d. If you do not drive over 65 miles per hour, then you will not get a speeding ticket.*

*e. Driving over 65 miles per hour is sufficient for getting a speeding ticket.*

*f. You get a speeding ticket, but you do not drive over 65 miles per hour.*

*g. Whenever you get a speeding ticket, you are driving over 65 miles per hour.*

**Solution:**

a. $\neg p$

b. $p \wedge \neg q$

c. $p \to q$

d. $\neg p \to \neg q$

e. $p \to q$

f. $q \wedge \neg p$

g. $p \to q$

**Question 10** *Let $p, q$ and $r$ be the propositions*

$p$ : *You get an A on the final exam*

$q$ : *You do every exercise in this book.*

$r$ : *You get an A in this class.*

*Write these propositions using $p, q,$ and $r$ and logical connectives.*

*a. You get an A in this class, but you do not do every exercise in this book.*

*b. You get an A on the final, you do every exercise in this book, and you get an A in this class.*

*c. To get an A in this class, it is necessary for you to get an A on the final.*

*d. You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.*

*e. Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.*

*f. You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.*

**Solution:**

a. $r \wedge \neg q$

b. $p \wedge q \wedge r$

c. $r \to p$

d. $p \wedge \neg q \wedge r$

e. $(p \wedge q) \to r$

f. $r \leftrightarrow (p \vee q)$

**Question 11** *Let $p$ and $q$ be the propositions*

*$p$ : Grizzly bears have been seen in the area.*

*q : Hiking is safe on the trail.*
*r : Berries are ripe along the trail.*

***Write these propositions using p and q and logical connectives.***

a. *Berries are ripe along the trail, but grizzly bears have not been seen in the area.*

b. *Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.*

c. *If berries are ripe along the trail, hiking is safe ifand only if grizzly bears have not been seen in the area.*

d. *It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.*

e. *For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.*

f. *Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.*

**Solution:**

a. $r \wedge \neg p$

b. $(\neg p \wedge q) \wedge r$

c. $(r \rightarrow q) \leftrightarrow \neg p$

d. $\neg q \wedge (\neg p \wedge r)$

e. $q \rightarrow (\neg r \wedge \neg p)$

f. $(p \wedge r) \rightarrow \neg q$

**Question 12** *(Pg.16, Q:18) Write each of these statements in the form "if p, then q" in English.*

a. *It is necessary to wash the boss's car to get promoted.*

b. *Winds from the south imply a spring thaw.*

c. *A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.*

d. *Willy gets caught whenever he cheats.*

e. *You can access the website only if you pay a subscription fee.*

f. *Getting elected follows from knowing the right people.*

g. *Carol gets seasick whenever she is on a boat.*

**Solution:**

a. If you want to be promoted, then you wash boss's car.

b. If winds are from the south, then spring thaws.

c. If you buy the computer less than a year ago, then the warranty holds good .

d. If Willy cheats, then he gets caught.

e. If you access the website then you pay a subscription fee.

f. If you know the right people, then you will get elected.

g. If Carol is on a boat, then she gets seasick.

**Question 13** *(Pg.16, Q:19) Write each of these statements in the form "if p, then q in English.*

a. *It snows whenever the wind blows from the northeast.*

b. *The apple trees will bloom if it stays warm for a week.*

c. *That the Pistons win the championship implies that they beat the Lakers.*

d. *It is necessary to walk 8 miles to get to the top of Long's Peak.*

e. *To get tenure as a professor, it is sufficient to be world-famous.*

f. *If you drive more than 400 miles, you will need to buy gasoline.*

g. *Your guarantee is good only if you bought your CD player less than 90 days ago.*

h. *Jan will go swimming unless the water is too cold.*

**Solution:**

a. If the wind blows from the northeast, then it snows.

b. If it stays warm for a week, then the apple trees will bloom.

c. If the Pistons win the championship, then they beat the Lakers.

d. If you get to the top of Long's Peak, then you must have walked 8 miles.

e. If you are world famous, then you will get tenure as a professor.

f. If you drive more than 400 miles, then you will need to buy gasoline.

g. If your guarantee is good, then you bought your CD player less than 90 days ago.

h. If the water is not too cold, then Jan will go swimming.

**Question 14** *(Pg.16, Q:20) Write each of these propositions in the form "p if and only if q" in English.*

*a. I will remember to send you the address only if you send me an e-mail message.*

*b. To be a citizen of this country, it is sufficient that you were born in the United States.*

*c. If you keep your textbook, it will be a useful reference in your future courses.*

*d. The Red Wings will win the Stanley Cup if their goalie plays well.*

*e. That you get the job implies that you had the best credentials.*

*f. The beach erodes whenever there is a storm.*

*g. It is necessary to have a valid password to log on to the server.*

*h. You will reach the summit unless you begin your climb too late.*

**Solution:**

a. If you send me an e-mail message, then I will remember to send you the address.

b. If you were born in the United States, then you are a citizen of this country.

c. If you keep your textbook, then it will be a useful reference in your future courses.

d. If their goaltender plays well, then the Red Wings will win the Stanley Cup.

e. If you get the job, then you had the best credentials.

f. If there is a storm, then the beach erodes.

g. If you log on to the server, then you have a valid password.

h. If you do not begin your climb too late, then you will reach the summit.

**Question 15** *(Pg.16, Q:21) Write each of these propositions in the form "p if and only if q" in English.*

*a. If it is hot outside you buy an ice cream cone, and if you buy an icecream cone it is hot outside.*

*b. For you to win the contest it is necessary and sufficient that you have the only winning ticket.*

*c. You get promoted only if you have connections, and you have connections only if you get promoted.*

*d. If you watch television your mind will decay, and conversely.*

*e. The trains run late on exactly those days when I take it.*

**Solution:**

a. You buy an ice cream cone if and only if it is hot outside.

b. You win the contest if and only if you have the only winning ticket.

c. You get promoted if and only if you have connections

d. Your mind will decay if and only if you watch television.

e. The trains run late if and only if I take it.

**Question 16** *(Pg.16, Q:22) Write each of these propositions in the form "p if and only if q"in English.*

*a. For you to get an A in this course, it is necessary and sufficient that you learn how to solve discrete mathematics problems.*

*b. If you read the newspaper every day, you will be informed, and conversely.*

*c. It rains if it is a weekend day, and it is a weekend day if it rains.*

*d. You can see the wizard only if the wizard is not in, and the wizard is not in only if you can see him.*

**Solution:**

a. You will get an $A$ in this course if and only if you learn how to solve discrete mathematics problems.

b. You will be informed if and only if you read the newspaper every day.

c. It rains if and only if it is a weekend day.

d. You can see the wizard if and only if he is not in.

**Question 17** *(Pg.16, Q:23) State the converse, contrapositive, and inverse of each of these conditional statements.*

*a. If it snows today, I will ski tomorrow.*

*b. I come to class whenever there is going to be a quiz.*

*c. If a positive integer is prime, then it has no divisors other than 1 and itself.*

**Solution:**

a.i. If I ski tomorrow, it snows today.

a.ii. If I won't ski tomorrow, then it will not/doesn't snows today.

a.iii. If it doesn't snow today, I will not ski tomorrow.

b.i. If I come to class, then there is going to be a quiz.

b.ii. If I don't come to class, then there is no quiz.

b.iii. If there is no quiz, then I don't come to class.

c.i. If a number has no divisors other than 1 and itself, then the number is prime

c.ii. If a number has a divisor other than 1 itself, then the number is not prime

c.iii. If a number is not prime, then it has a divisor other than 1 and itself.

**Question 18** *(Pg.16, Q:24) State the converse, contrapositive, and inverse of each of these conditional statements.*

*a. If it snows tonight, then I will stay at home.*

*b. I go to the beach whenever it is a sunny summer day.*

*c. When I stay up late, it is necessary that I sleep until noon.*

**Solution:**

a. (i) Converse: If I stay home, then it will snow tonight.
(ii) Contrapositive: If I do not stay at home, then it will not snow tonight.
(iii) Inverse: If it does not snow tonight, then I will not stay home.

b. (i) Converse: Whenever I go to the beach, it is a sunny summer day.
   (ii) Contrapositive: Whenever I do not go to the beach, it is not a sunny summer day.
   (iii) Inverse: Whenever it is not a sunny day, I do not go to the beach.

c. (i) Converse: If I sleep until noon, then I stayed up late.
   (ii) Contrapositive: If I do not sleep until noon, then I did not stay up late.
   (iii) Inverse: If I don't stay up late, then I don't sleep until noon.

**Question 19** *(Pg.16, Q:42) Is the assertion "This statement is false"a proposition* ?

**Solution:**
This assertion not a proposition (Since the truth value is neither TRUE nor FALSE). This is a self-contradictory statement. Assigning TRUE to this statement implies that its truth value is FALSE and vice versa. This is a paradox.

**Question 20** *(Pg.16, Q:43) The nth statement in a list of 100 statements is "Exactly n of the statements in this list are false."*

a. *What conclusions can you draw from these statements?*

b. *Answer part (a) if the nth statement is "At least n of the statements in this list are false."*

c. *Answer part (b) assuming that the list contains 99 statements.*

**Solution:**

a. The 99th statement is true and the rest are false.

b. Statements 1 through 50 are all true and statements 51 through 100 are all false.

c. This can not happen; it is a paradox, showing that these cannot be statements.

**Question 21** *(Pg.16, Q:44) An ancient Sicilian legend says that the barber in a remote town who can be reached only by traveling a dangerous mountain road shaves those people, and only those people, who do not shave themselves. Can there be such a barber?*

**Solution:**
Converting the paradox into a logical sentence,
$N(y)$ : y do not shave by himself.
$S(x,y)$ : x shaves y.
$\exists x\ (\forall y\ (N(y) \leftrightarrow S(x,y)))$
If UOD is set of all human beings, a female barber x exists and truth value is TRUE.
If UOD is set of all males and they shaves, then who will shave the barber. If he shaves himself, then sufficiency is violated. If x do not shave himself, then barber will shave him. This violates necessay and hence such a barber cannot exist.

**Question 22** *(Pg.16, Q:45) Each inhabitant of a remote village always tells the truth or always lies. A villager will only give a "Yes" or a "No" response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a fork in the road. One branch leads to the ruins you want to visit; the other branch leads deep into the jungle. A villager is standing at the fork in the road. What one question can you ask the villager to determine which branch to take?*

**Solution:**
Ask him "If i am asking you whether road 1 leads to ruins, then what would be your answer ?". Note the if he were a person who always tells truth, then tourist can follow his words. If he were a person who always lies, then he would have negated his answer twice and hence, tourist can follow his words.

**Question 23** *(Pg.16, Q:46) An explorer is captured by a group of cannibals. There are two types of cannibals-those who always tell the truth and those who always lie. The cannibals will barbecue the explorer unless he can determine whether a particular cannibal always lies or always tells the truth. He is allowed to ask the cannibal exactly one question.*
*a) Explain why the question "Are you a liar?" does not work.*
*b) Find a question that the explorer can use to determine whether the cannibal always lies or always tells the truth.*

**Solution:**
a) Note that the answer given by all cannibals are "NO", and therefore we cannot distinguish a liar from honest cannibals.
b) Question to be asked should be a hypothetical question. "If i am asking you whether you are a liar, what will be your response ?". Observe that if the cannibal is a honest cannibal, then he answers "NO", otherwise "YES".

**Exercises 55-59** relate to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, $A$ and $B$. Determine, if possible, what $A$ and $B$ are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusions?

**Question 24** *(Pg.16, Q:55) A says "At least one of us is a knave" and B says nothing.*

**Solution:**
All possibilities for $A$ and $B$ are given along with the truth value of the above statement.

| A | B | Truth value of A's stmt | Is assertion possible? |
|---|---|---|---|
| knight | knight | FALSE | No |
| knight | knave | FALSE | Yes |
| knave | knight | TRUE | No |
| knave | knave | TRUE | No |

$A$ Knave cannot say he is a knave, so $A$ is a Knight and his statement is true. Therefore $B$ is a Knave.

**Question 25** *(Pg.16, Q:56)A says "The two of us are both knights"and B says "A is a knave".*

**Solution:**
If $A$ is a knight, then his statement that both of them are knights is true, and both will be telling the truth. But that is impossible, because $B$ is asserting otherwise (that $A$ is a knave). If $A$ is a knave, then $B$'s assertion is true, so he must be a knight, and $A$'s assertion is false, as it should be. Thus we conclude that $A$ is a knave and $B$ is a knight.

**Question 26** *(Pg.16, Q:57)A says "I am a knave or B is a knight"and B says nothing.*

**Solution:**
If $A$ is a knave, then we get an immediate contradiction, since this implies, $A$ is a knight and $B$ is a knave, which is impossible. Hence, $A$ must be a knight, then his statement that $B$ is a knight is true. Thus we conclude that $A$ is a knight and $B$ is also a knight.

**Question 27** *(Pg.16, Q:58)Both A and B say "I am a knight."*

**Solution:**
All possibilities for $A$ and $B$ are given along with the truth value of the above statement.

| $A$ | $B$ | Truth value of stmts for $A$ and $B$ | Is assertion possible? |
|---|---|---|---|
| knight | knight | `A(T), B(T)` | Yes |
| knight | knave | `A(T), B(F)` | Yes |
| knave | knight | `A(F), B(T)` | Yes |
| knave | knave | `A(F), B(T)` | Yes |

It is possible for either $A$ or $B$ to be either a knight or a knave.

**Question 28** *(Pg.16, Q:59) A says "We are both knaves" and B says nothing.*

**Solution:**
All possibilities for A and B are given along with the truth value of the above statement.

| A | B | Truth value of A's stmt |
|---|---|---|
| knight | knight | FALSE |
| knight | knave | FALSE |
| knave | knight | TRUE |
| knave | knave | FALSE |

Therefore A is Knave and B is Knight.
**Exercises 60-65** are puzzles that can be solved by translating statements into logical expressions and reasoning from these expressions using truth tables.

**Question 29** *(Pg.16, Q:60) The police have three suspects for the murder of Mr. Cooper: Mr. Smith, Mr. Jones, and Mr. Williams. Smith, Jones, and Williams each declare that they did not kill Cooper. Smith also states that Cooper was a friend of Jones and that Williams disliked him. Jones also states that he did not know Cooper and that he was out of town the day Cooper was killed. Williams also states that he saw both Smith and Jones with Cooper the day of the killing and that either Smith or Jones must have killed him. Can you determine who the murderer was if*

a. *one of the three men is guilty, the two innocent men are telling the truth, but the statements of the guilty man may or may not be true?*

b. *innocent men do not lie?*

**Solution:**

a. We look at the three possibilities of who the innocent men might be. If Smith and Jones are innocent (and therefore telling the truth), then we get an immediate contradiction, since

Smith said that Jones was a friend of Cooper, but Jones said that he did not even know Cooper. If Jones and Williams are the innocent truth-tellers, then we again get a contradiction, since Jones says that he did not know Cooper and was out of town, but Williams says he saw Jones with Cooper (presumably in town, and presumably if we was with him, then he knew him). Therefore it must be the case that Smith and Williams are telling the truth. Their statements do not contradict each other. Based on Williams statement, we know that Jones is lying, since he said that he did not know Cooper when in fact he was with him. Therefore Jones is the murderer.

b. If all are innocent, then all of their statement is true, but that is impossible. Thus there is a possibility where more than one are guity.

**Question 30** *(Pg.16, Q:61) Steve would like to determine the relative salaries of three coworkers using two facts. First, he knows that if Fred is not the highest paid of the three, then Janice is. Second, he knows that if Janice is not the lowest paid, then Maggie is paid the most. Is it possible to determine the relative salaries of Fred, Maggie, and Janice from what Steve knows? If so, who is paid the most and who the least? Explain your reasoning.*

**Solution:**
In the order of decreasing salary: Fred, Maggie, Janice.

**Question 31** *(Pg.16, Q:62) Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Explain your reasoning.*

**Solution:**
Heather, Abby and Randy are not chatting. Vijay and Kevin are chatting.

**Question 32** *(Pg.16, Q:63) A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded that if the butler is telling the truth then so is the cook; the cook and the gardener cannot both be telling the truth; the gardener and the handyman are not both lying; and if the handyman is telling the truth then the cook is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning.*

**Solution:**
If the cook is telling truth, then the gardener is lying. This implies, handyman is telling truth, who says cook is lying, which is a contradiction. Thus, cook is lying. If the butler is telling truth, then cook is also telling truth. This implies, gardener is lying and handyman is telling truth. But handyman says, cook is lying, which is a contradiction. Thus, butler is lying. But we cannot determine whether the gardener is telling the truth or whether the handyman is telling the truth.

**Question 33** *(Pg.16, Q:64) Four friends have been identified as suspects for an unauthorized access into a computer system. They have made statements to the investigating authorities. Alice said "Carlos did it." John said "I did not do it." Carlos said "Diana did it." Diana said "Carlos lied when he said that I did it."*

*a. If the authorities also know that exactly one of the four suspects is telling the truth, who did it? Explain your reasoning.*

*b. If the authorities also know that exactly one is lying, who did it? Explain your reasoning.*

**Solution:**

a. John only did it. There are four cases to consider. If Alice is the sole truth-teller, then Carlos did it; but this means that John is telling the truth, a contradiction. If John is the sole truth-teller, then Diana must be lying, so she did it, but then Carlos is telling the truth, a contradiction. If Carlos is the sole truth-teller, then Diana did it, but that makes John truthful, again a contradiction. So the only possibility is that Diana is the sole truth-teller. This means that John is lying when he denied it, so he did it. Note that in this case both Alice and Carlos are indeed lying.

b. Again there are four cases to consider. Since Carlos and Diana are making contradictory statements, the liar must be one of them (we could have used this approach in part (a) as well). Therefore Alice is telling the truth, so Carlos did it. Note that John and Diana are telling the truth as well here, and it is Carlos who is lying.

**Question 34** *(Pg.16, Q:65) Solve this famous logic puzzle, attributed to Albert Einstein, and known as the zebra puzzle. Five men with different nationalities and with different jobs live in consecutive houses on a street. These houses are painted different colors. The men have different pets and have different favorite drinks. Determine who owns a zebra and whose favorite drink is mineral water (which is one of the favorite drinks) given these clues: The Englishman lives in the red house. The Spaniard owns a dog. The Japanese man is a painter. The Italian drinks tea. The Norwegian lives in the first house on the left. The green house is immediately to the right of the white one. The photographer breeds snails. The diplomat lives in the yellow house. Milk is drunk in the middle house. The owner of the green house drinks coffee. The Norwegian's house is next to the blue one. The violinist drinks orange juice. The fox is in a house next to that of the physician. The horse is in a house next to that of the diplomat. [Hint: Make a table where the rows represent the men and columns represent the color of their houses, their jobs, their pets, and their favorite drinks and use logical reasoning to determine the correct entries in the table.]*

**Solution:**

| Num. from left | Nationality | H.color | Job | Pet | Fav Drink |
|---|---|---|---|---|---|
| 1 | Norwegian | Yellow | Diplomat | Fox | **Mineral water** |
| 2 | Italian | Blue | Physician | Horse | Tea |
| 3 | Englishman | Red | Photographer | Snail | milk |
| 4 | Spaniard | White | Violinist | Dog | Orange juice |
| 5 | Japanese | Green | Painter | **Zeebra** | Coffee |

**Question 35** *(Pg.28, Q:23) Show that* $(p \to r) \wedge (q \to r)$ *and* $(p \vee q) \to r$ *are logically equivalent.*

**Solution:**

| $p$ | $q$ | $r$ | $p \to r$ | $q \to r$ | $p \vee q$ | $(p \to r) \wedge (q \to r)$ | $(p \vee q) \to r$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | F | F | T | F | F |
| T | F | T | T | T | T | T | T |
| T | F | F | F | T | T | F | F |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | T | F | F |
| F | F | T | T | T | F | T | T |
| F | F | F | T | T | F | T | T |

Hence, $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.

**Question 36** *(Pg.28, Q:24) Show that* $(p \to q) \vee (p \to r)$ *and* $(p \to (q \vee r))$ *are logically equivalent.*

**Solution:**
L.H.S : $(p \to q) \vee (p \to r)$
$\leftrightarrow (\neg p \vee q) \vee (\neg p \vee r)$
$\leftrightarrow (\neg p \vee \neg p) \vee (q \vee r)$ (By Associativity property)
$\leftrightarrow \neg p \vee (q \vee r)$ (by the property $(p \vee p \to p)$)
$\leftrightarrow (p \to (q \vee r))$
Hence proved.
**Note: It can be proved using truth table method also.**

**Question 37** *(Pg.28, Q:25) Show that* $(p \to r) \vee (q \to r)$ *and* $(p \wedge q) \to r$ *are logically equivalent.*

**Solution:**

| p | q | r | $p \to r$ | $q \to r$ | $(p \to r) \vee (q \to r)$ | $p \wedge q$ | $(p \wedge q) \to r$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | F | F | F | T | F |
| T | F | T | T | T | T | F | T |
| T | F | F | F | T | T | F | T |
| F | T | T | T | T | T | F | T |
| F | T | F | T | F | T | F | T |
| F | F | T | T | T | T | F | T |
| F | F | F | T | T | T | F | T |

**Question 38** *(Pg.28, Q:26) Show that* $\neg p \to (q \to r)$ *and* $q \to (p \vee r)$ *are logically equivalent.*

**Solution:**

| $p$ | $q$ | $r$ | $\neg p$ | $q \to r$ | $p \vee r$ | $\neg p \to (q \to r)$ | $q \to (p \vee r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | T | T | T | T |
| T | T | F | F | F | T | T | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | F | F | F |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | F | T | T |

Hence, $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

**Question 39** *(Pg.28, Q:27) Show that* $p \leftrightarrow q$ *and* $(p \to q) \wedge (q \to p)$ *are logically equivalent.*

**Solution:**

| $p$ | $q$ | $p \to q$ | $q \to p$ | $(p \to q) \wedge (q \to p)$ | $p \leftrightarrow q$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | T | F | F |
| F | T | T | F | F | F |
| F | F | T | T | T | T |

**Question 40** *(Pg.28, Q:28) Show that* $p \leftrightarrow q$ *and* $\neg p \leftrightarrow \neg q$ *are logically equivalent.*

**Solution:**

| p | q | $p \leftrightarrow q$ | $\neg p$ | $\neg q$ | $\neg p \leftrightarrow \neg q$ |
|---|---|---|---|---|---|
| T | T | T | F | F | T |
| T | F | F | F | T | F |
| F | T | F | T | F | F |
| F | F | T | T | T | T |

**Question 41** *(Pg.28, Q:29) Show that* $(p \to q) \wedge (q \to r) \to (p \to r)$ *is a tautology.*

**Solution:**

| $p$ | $q$ | $r$ | $p \to q$ | $q \to r$ | $(p \to q) \wedge (q \to r)$ | $p \to r$ | $(p \to q) \wedge (q \to r) \to (p \to r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | F | T | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | F | T | T |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T |

Hence, $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

**Question 42** *(Pg.28, Q:30) Show that* $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ *is a tautology.*

**Solution:**

| $p$ | $q$ | $r$ | $p \vee q$ | $\neg p$ | $(\neg p \vee r)$ | $(p \vee q) \wedge (\neg p \vee r)$ | $(q \vee r)$ | $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ |
|---|---|---|---|---|---|---|---|---|
| F | F | F | F | T | T | F | F | T |
| F | F | T | F | T | T | F | T | T |
| F | T | F | T | T | T | T | T | T |
| F | T | T | T | T | T | T | T | T |
| T | F | F | T | F | F | F | F | T |
| T | F | T | T | F | T | T | T | T |
| T | T | F | T | F | F | F | T | T |
| T | T | T | T | F | T | T | T | T |

**Question 43** *(Pg.28, Q:31) Show that* $(p \to q) \to r$ *and* $p \to (q \to r)$ *are not logically equivalent.*

**Solution:**

| p | q | r | $p \to q$ | $(p \to q) \to r$ | $q \to r$ | $p \to (q \to r)$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | T | F | F | F |
| T | F | T | F | T | T | T |
| T | F | F | F | T | T | T |
| F | T | T | T | T | T | T |
| F | T | F | T | F | F | T |
| F | F | T | T | T | T | T |
| F | F | F | T | F | T | T |

**Question 44** *(Pg.28, Q:32) Show that* $(p \land q) \to r$ *and* $(p \to r) \land (q \to r)$ *are not equivalent.*

**Solution:**

| $p$ | $q$ | $r$ | $p \land q$ | $p \to r$ | $(q \to r)$ | $(p \to r) \land (q \to r)$ | $(p \land q) \to r$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | F |
| T | F | T | F | T | T | T | T |
| T | F | F | F | F | T | F | T |
| F | T | T | F | T | T | T | T |
| F | T | F | F | T | F | F | T |
| F | F | T | F | T | T | T | T |
| F | F | F | F | T | T | T | T |

Hence, $(p \land q) \to r$ and $(p \to r) \land (q \to r)$ are not equivalent.

**Question 45** *(Pg.28, Q:33) Show that* $(p \to q) \to (r \to s)$ *and* $(p \to r) \to (q \to s)$ *are not logically equivalent.*

**Solution:**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| $p$ | $q$ | $r$ | $s$ | $p \to q$ | $r \to s$ | $3 \to 4$ | $p \to r$ | $q \to s$ | $5 \to 6$ |
| T | T | T | T | T | T | T | T | T | T |
| T | T | T | F | T | F | F | T | F | F |
| T | T | F | T | T | T | T | F | T | T |
| T | T | F | F | T | T | T | F | F | T |
| T | F | T | T | F | T | T | T | T | T |
| T | F | T | F | F | F | T | T | T | T |
| T | F | F | T | F | T | T | F | T | T |
| T | F | F | F | F | T | T | F | T | T |
| F | T | T | T | T | T | T | T | T | T |
| F | T | T | F | T | F | F | T | F | F |
| F | T | F | T | T | T | T | T | T | T |
| F | T | F | F | T | T | **T** | T | F | **F** |
| F | F | T | T | T | T | T | T | T | T |
| F | F | T | F | T | F | **F** | T | T | **T** |
| F | F | F | T | T | T | T | T | T | T |
| F | F | F | F | T | T | T | T | T | T |