Assignment - FOL - Solutions

1. Write the following in first order logic.

   (a) Some boys are sharp and intelligent.
   $UOD(x)$: all persons.
   $Sharp(x)$: x is sharp.
   $Boy(x)$: x is a boy.
   $Intelligent(x)$: x is intelligent.

   $\exists x(Sharp(x) \wedge Boy(x) \wedge Intelligent(x))$

   (b) Not all boys are intelligent.

   $\neg\forall x(Boy(x) \rightarrow Intelligent(x))$
   or $\exists x(Boy(x) \wedge \neg Intelligent(x))$

   (c) Some students of DM course has cleared JEE main and the rest cleared SAT.
   $UOD(x)$: all persons.
   $ClearJEE(x)$: x clears JEE main.
   $ClearSAT(x)$: x clears SAT.

   $\exists x[Stud(x) \wedge ClearJEE(x) \wedge \forall y(y \neq x \rightarrow ClearSAT(y))]$

   (d) Some thing that is white is not always milk, whereas the milk is always white.
   $UOD(x)$: things.
   $White(x)$: x is white.
   $Milk(x)$: x is milk.

   $\exists x(White(x) \wedge \neg Milk(x)) \wedge \forall x(Milk(x) \rightarrow White(x))$

   (e) Unless you report to the exam cell before 9 am, you will not be permitted to write quizzes. - Also, write inverse, converse and contrapositive.
   Let P : You report to the exam cell before 9am.
   Q : You will be permitted to write quizzes.
   Therefore the statement is $\neg P \rightarrow \neg Q$
   Converse: $\neg Q \rightarrow \neg P$
   If you are not permitted to write quizzes, then you have not reported to the exam cell before 9am.
   Inverse: $P \rightarrow Q$
   If you report to the exam cell before 9am, then you will be permitted to write quizzes.
   Contrapositive: $Q \rightarrow P$
   If you are permitted to write quizzes, then you reported to the exam cell before 9am.

   (f) Breakfast is served in mess on all days between 7am and 9am except Sunday. And, on Sundays it is served till 9.15 am.
   $UOD(x)$: days.
   $Day(x)$: x is a weekday.
   Breakfast-time-non-sunday(x): Breakfast is served in mess on x between 7am and 9am.
   Breakfast-time-sunday(x): Breakfast is served in mess on x till 9.15am.

$\forall x[(Day(x) \wedge x \neq \text{SUNDAY}) \rightarrow \text{Breakfast-time-non-sunday}(x)] \wedge \forall x((x = \text{SUNDAY}) \rightarrow \text{Breakfast-time-sunday}(x))$

(g) The speed of light is not same in all mediums. The speed of light in fiber is 2×10^8 m/s. Therefore, there exists at least two mediums having different speed of light.
$UOD(x)$: mediums.
$Medium(x)$: Light travels in medium x.
$Speed(x)$: Speed of light in medium x.
P: Speed of light in fiber is 2×10^8 m/s.

$\neg\forall x\forall y[(Medium(x) \wedge Medium(y)) \rightarrow Speed(x) = Speed(y)] \quad \wedge \quad P$
Conclusion:
$\exists x\exists y[Medium(x) \wedge Medium(y) \wedge (x \neq y) \wedge Speed(x) \neq Speed(y)]$

(h) Some students have joined IIITDM. There exists a student who has not joined any IIITDM. Not all students have cleared JEE advanced. Therefore, some students have joined deemed universities.
$UOD(x)$: people.
$UOD(y)$: Educational institutes.
$Stud(x)$: x is a student.
$IIITDM(y)$: y is a IIITDM.
$JoinIIITDM(x, y)$: x joins IIITDM y.
$ClearJEE(x)$: x cleared JEE advanced.
$JoinDeemed(x)$: x joins a deemed university.

$\exists x[Stud(x) \wedge \exists y(IIITDM(y) \wedge JoinIIITDM(x, y))]$
$\exists x[Stud(x) \wedge \forall y(IIITDM(y) \rightarrow \neg JoinIIITDM(x, y))] \quad \wedge$
$\neg\forall x(Stud(x) \rightarrow ClearJEE(x))$
Conclusion:
$\exists x(Stud(x) \wedge JoinDeemed(x))$.

(i) Each student of IIITDM likes either carnatic music or Hindustani music. Further, some students are athletes, however, they have not represented the institute in inter sports meet. Carnatic music is a form of meditation. Music club at IIITDM is not that active. Therefore, students of IIITDM are neither good in sports nor in music.
$uod(x)$: people
$Stud(x)$: x is a student.
$C(x)$: x likes carnatic music.
$H(x)$: x likes Hindustani music.
$Athlete(x)$: x is an athlete.
$notrep(x)$: x has not represented the institute in inter sports meet.
CM : Carnatic music is a form of meditation
MA : Music club at IIITDM is active
$S(x)$: x is good in sports.
$M(x)$: x is good in music.

$\forall x[Stud(x) \rightarrow \neg(C(x) \leftrightarrow H(x))] \quad \wedge$
$\exists x[Stud(x) \wedge Athlete(x) \wedge notrep(x)] \quad \wedge$
CM $\wedge$
$\neg$ MA $\wedge$

Conclusion:

$\forall x(Stud(x) \rightarrow \neg S(x) \wedge \neg M(x))$.

(j) Show that in a group of five people (where any two people are either friends or enemies), there are not necessarily three mutual friends or three mutual enemies.
$People(x) : x$ is a person.
$MutualFriends(x, y, z) : x, y, z$ are mutual friends.
$MutualEnemies(x, y, z) : x, y, z$ are mutual enemies.

$\neg\forall y_1, y_2, y_3, y_4, y_5[(y_i \neq y_j, 1 \leq i < j \leq 5 \ \wedge \ \forall_{1 \leq k \leq 5} \ People(y_k)) \rightarrow$
$\{\exists_{1 \leq u,v,w \leq 5}((u \neq v \neq w) \ \wedge \ MutualFriends(y_u, y_v, y_w)) \vee$
$\exists_{1 \leq u,v,w \leq 5}((u \neq v \neq w) \ \wedge \ MutualEnemies(y_u, y_v, y_w))\} \ ]$

(k) Show that in a group of 10 people (where any two people are either friends or enemies), there are either three mutual friends or four mutual enemies, and there are either three mutual enemies or four mutual friends.
$People(x) : x$ is a person.
$Mutual3Friends(x, y, z) : x, y, z$ are mutual friends.
$Mutual3Enemies(x, y, z) : x, y, z$ are mutual enemies.
$Mutual4Friends(w, x, y, z) : w, x, y, z$ are mutual friends.
$Mutual4Enemies(w, x, y, z) : w, x, y, z$ are mutual enemies.

$\exists y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}[y_i \neq y_j, 1 \leq i < j \leq 10 \ \wedge \ \forall_{1 \leq k \leq 10} \ People(y_k) \wedge$
$\exists_{1 \leq u,v,w,x \leq 10}\{(u \neq v \neq w \neq x) \wedge \ (Mutual3Friends(y_u, y_v, y_w) \oplus Mutual4Enemies(y_u, y_v, y_w, y_x)) \ \} \wedge$
$\exists_{1 \leq u,v,w,x \leq 10}\{(u \neq v \neq w \neq x) \wedge \ (Mutual3Enemies(y_u, y_v, y_w) \oplus Mutual4Friends(y_u, y_v, y_w, y_x)) \ \} \ ]$

2. Are there statements in English that can not be expressed using FOL. Mention at least one, if it exists. Statements that need quantification of sets.
For example: There exist a subset of students in DM class having i-phones.

If one attempts FOL for the above statement, then the expression goes like this;
$\exists S(S \subset UOD \wedge \forall x(S(x) \wedge stud(x) \rightarrow iphone(x))$
However, FOL does not allow quantification with respect to subsets, it only allows quantification with respect to elements in UOD (atomic variables). The above expression comes under the next level logic, namely, second order logic.

3. `Puzzle to assess Logical Reasoning ability:` Five persons A, B, C, D, E are in a compartment in a train. A, C, E are men and B, D are women. The train passes through a tunnel and when it emerges, it is found that E is murdered. An inquiry is held, A, B, C, D make the following statements. Each makes two statements.

A says: I am innocent. B was talking to E when the train was passing through the tunnel.
B says: I am innocent. I was not talking to E when the train was passing through the tunnel.
C says: I am innocent. D committed the murder.
D says: I am innocent. One of the men committed the murder.

Out of 8 statements given above, 4 are true and 4 are false. Who is the murderer. Support your answer with a precise and concise justification. Note: Each person is making exactly two statements.

Solution: Let the statements given by A be A_1, A_2 in order. Similarly, let us denote $B_i, C_i, D_i, i \in \{1, 2\}$ for other statements. Since one of A, B, C, D did the murder, three among A_1, B_1, C_1, D_1 are `true`.

Observe that the statements A_2, B_2 are inverse to each other. Therefore one of them is true. It follows that the four true statements are from $A_1, B_1, C_1, D_1, A_2, B_2$. Hence, we could conclude that C_2, D_2 are `false`. This implies, D did not commit the murder and none of the men committed the murder. Further, out of the two women B, D; since D has not committed the murder, we conclude that B is the murderer.